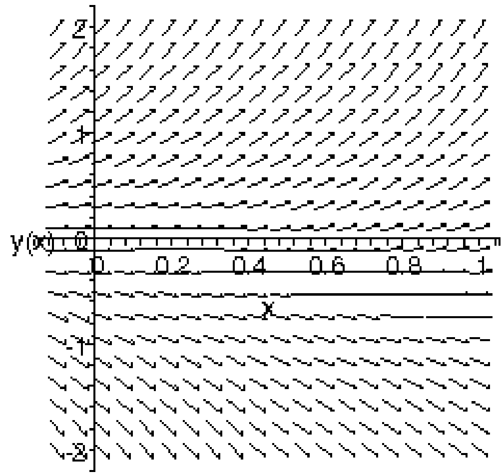
SM222 Differential Equations with Matrices
Final Exam   Fall 2003

Show all work. Short Answers: 4 points each

1. A 120 gallon tank initially contains 90 pounds of salt dissolved in 90 gallons of water. Brine with a concentration of 2 lb/gal flows into the tank at a rate of 4 gal/min and the well stirred mixture flows out at a rate of 3 gal/min. Setup the differential equation for the amount, x, of salt in the tank at any time, t.

2. For the direction field in figure #1, the Euler method approximation for y(0.5) with initial condition y(0) = 1 and step size h = 0.5 is: (show all work)



3. For the initial value problem $y' = x + y,\ \ y(0) = 1$, the improved Euler approximation to $y(0.1)$ with h = 0.1 is:

4. For an LRC series circuit with L = 2 henries, R = 14 ohms, C = 0.05 farad, and an electromotive force = 120 volts, the steady state charge on the capacitor is:

5. The appropriate UC guess for the particular solution to $x'' + x = 4\cos(t)$ is $x_p = ?$

6. The appropriate variation of parameter guess for the particular solution to $x'' + x = 4\cos(t)$ is $x_p = ?$

7. A mass weighing 100 pounds is attached to a spring which is stretched 2 inches before coming to equilibrium (think feet). An external force of $5\cos(wt)$ acts on the mass. Ignoring damping, what value of w will give natural resonance? Use $g = 32$.

8. A damped mass-spring system without an external force is described by $2x'' + \beta x' + 8x = 0$. The system is critically damped for what value of β?

9. In the Fourier series for $f(x) = 5x,\ -\pi \le x \le \pi$, which $a_n = 0$? Which b_n?

10. How do the boundary values for the zero-ends heat experiment differ from the insulated-ends experiment?

Long Answers: 10 points each

11. Solve $\cos(x)\dfrac{dy}{dx}+\sin(x)y=\cos^2(x), \quad y(0)=-1$.

12. Use the method of undetermined coefficients to solve:

$$y''+2y'+y=2-2\sin x,\ \ y(0)=0,\ \ y'(0)=1$$

13. Use eigenvalues and vectors to solve:

$$\frac{dx}{dt}=3x+2y$$
$$\frac{dy}{dt}=-2x+3y$$
$$x(0)=2,\ \ y(0)=0$$

14. Given $y=f(x)=6x^2, 0\le x\le 4,$ we wish to extend the function to $-4\le x\le 4$ so as to obtain a Fourier cosine series.
   a. Graph the extended function form −4 to 4.
   b. Find the general formula for b_n.
   c. Find the the value of a_0 and the general formula for a_n.
   d. Write out the series giving at least the first three nonzero terms.

15. A thin bar of length 4 with constant of diffusivity = 1 initially has a temperature distribution = $6x^2$. Its ends are insulated for times > 0. Mathematically:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$$
$$\frac{\partial u}{\partial x}(0,t)=0,\ \ \frac{\partial u}{\partial x}(4,t)=0,\ \ t>0$$
$$u(x,0)=6x^2,\ \ 0\le x\le 4$$

   a. Find the function for the temperature, $u(x,t)$. Use separation of variables and show all steps. (Hint: see #14). Do not assume case 3.
   b. What is the steady state value of the temperature ($t\to+\infty$)?

16. Write the three loop equations (identifying which is which) for the electrical network below in terms of i_3 and q_2. Start by giving the current equation.